AIAA JOURNAL
Vol. 33, No. 12, December 1995

Multigrid Approach to Overset Grid Communication

Rudy A. Johnson* and Dave M. Belk†
U.S. Air Force Wright Laboratory, Eglin Air Force Base, Florida 32542-6810

The nonaligned multigrid method is introduced as an alternative to boundary point interpolation for communication in overset grid systems. The nonaligned multigrid method treats the individual grids as levels in a multigrid solution procedure. This technique was able to converge 69% faster than standard boundary point interpolation for Mach 2 flow over a 5-deg ramp and 28% faster for a more complex reflected shock case. For these test cases, the largest increase in convergence rate came from applying the nonaligned multigrid method to the communication between overset grids. Further coarsening of the global grid was next in effectiveness, and coarsening of local grids gave little improvement.

Introduction

THE usual approach to communication between overset grids[1–4] is through boundary point interpolation. The solution is interpolated at grid outer boundaries and at fringe points around holes. Holes are created in grids by eliminating points that either lie inside a solid body or are considered inferior to points in an overlapping grid. This approach leads to a method of solution known as a Schwartz alternating method.[4,5] A disadvantage of Schwartz' method is that in the region of overlap there are multiple solutions, none of which are considered more correct than others. Also, the work expended to obtain these multiple solutions is wasted on all but one of the grids. Because of this, the usual strategy for overset grid systems is to minimize the region of overlap.

The purpose of this study is to show the feasibility of using a nonaligned multigrid technique[6] to communicate between overset grids instead of boundary point interpolation. Instead of allowing each grid to have its own different solution, only one grid is considered the "top level" grid. Interpolation of both residuals and dependent variables from the top level is used to calculate a defect correction driving the solution on other grids in the region of overlap. This technique makes use of all grid points in the overlap regions of an overset grid system. As will be demonstrated later, one benefit is an improved convergence rate for the overset grid system.

The nonaligned multigrid can be thought of as the generalized application of the full approximation scheme[7–9] to an overset grid system or as a local refinement technique[9–12] with the local levels generated independently of the global level. Previous researchers have applied multigrid to enhance convergence of overset grid systems,[13–15] but all of these studies confined the application of multigrid to component grids. Communication between overset grids was via boundary point interpolation. The approach here is to replace boundary point interpolation with multigrid style communication.

The nonaligned multigrid technique is described in more detail later, and two test cases are presented to illustrate the method. A 5-deg ramp and a 5-deg ramp near a flat plate, both at Mach 2.0, are solved using a first-order explicit flux difference splitting[16,17] approximation of the two-dimensional Euler equations for inviscid compressible flow.

Presented as Paper 93-0769 at the AIAA 31st Aerospace Sciences Meeting, Reno, NV, Jan. 11–14, 1993; received July 1, 1994; revision received March 20, 1995; accepted for publication March 22, 1995.

*Aerospace Engineer, Computational Fluid Dynamics Section, Aerodynamics Branch, Weapon Flight Mechanics Division, Armament Directorate. Member AIAA.

†Senior Research Scientist, Computational Fluid Dynamics Section, Aerodynamics Branch, Weapon Flight Mechanics Division, Armament Directorate. Associate Fellow AIAA.

Nonaligned Multigrid

The idea here is to treat the overlapping grids as independent levels of a multigrid full approximation scheme (FAS) procedure. By doing this the boundary interpolations of the dependent variables in the Schwartz method are replaced by interpolation of the dependent variables, a defect correction, and a lower level correction over the entire overlap region in the nonaligned multigrid (NAM) method. The defect correction stops grid cells of different sizes from degrading the global solution and allows the previously wasted points to be effectively used in the solution procedure.

NAM is a generalized local refinement method. The form of the NAM equations are essentially the same as the FAS equations with the variations coming about in the interpolation (restriction and prolongation operators) and information passing requirements.

The NAM solution procedure is obtained by making two modifications to the FAS procedure. First, a loop over the number of lower and higher levels is added for the restriction and prolongation stages, respectively. This allows the current level to restrict information to any or all lower levels and to prolongate to any or all higher levels.

The second modification is due to the arbitrary orientations permissible with NAM. Similar problems occur when applying multigrid to unstructured meshes.[18] A more general set of interpolation operators than typical in the FAS procedure must be used. Bilinear interpolation is used for both restriction and prolongation operators. This requires the residuals to be treated as point quantities; hence the volume is removed on the higher level and restored on the lower level after the restriction.[6]

The communications between levels can be illustrated by applying the NAM procedure to the system of overset grids depicted in part a of Fig. 1. With the levels as indicated, the flow solver is applied to level 1, followed by the generation of the defect correction. Level 1 then restricts the dependent variables and defect correction to level 2 in region 1 and to the portion of level 3 directly under the remainder of level 1. The flow solver is applied to level 2, and the defect correction is generated. The entirety of level 2 then restricts information to level 3.

The flow solver is applied to level 3, and the lower level correction is generated. Level 3 then prolongates the lower level correction to all of level 2 and to the portion of level 1 not in region 1. The flow solver can then be applied on level 2, and a lower level correction is generated on level 2 in region 1. Level 2 prolongates this correction to level 1 in region 1. The last step is the application of the flow solver on level 1 and the decision to repeat the cycle or to stop. The grid schedule analogous to the FAS V-Cycle diagram is shown in part b of Fig. 1 for the three level treatment of grids just described (ν_i indicates the number of iterations on level i).

For faster convergence of this problem any of the grids may be coarsened and added as additional levels. Care should be used when coarsening levels other than the lowest level. For example, coarsening level 1 and inserting it as level 2 may result in a grid with worse resolution than the level receiving the restricted information.

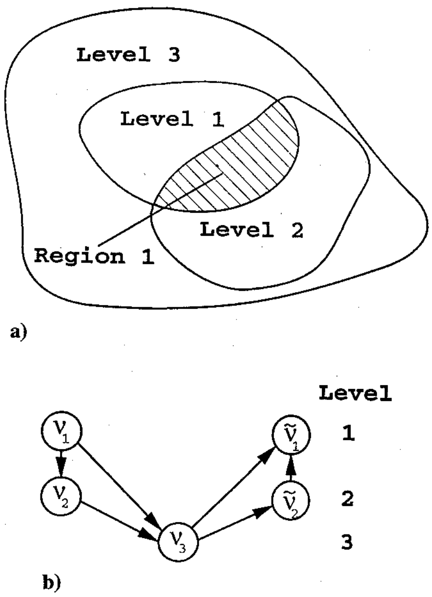

Fig. 1 NAM three level a) system and b) grid schedule.

This would result in forcing lower resolution onto the lower levels. The lowest level (typically the global grid) is usually the safest to coarsen. Coarsening the global grid will in general provide faster convergence by allowing larger time steps for a more rapid development of the flow structure.

Results

The governing equations are the strong conservation law form of the nondimensionalized Euler equations in two dimensions written in terms of a body-conforming curvilinear coordinate system. These equations are discretized using a first-order explicit flux difference splitting (FDS) algorithm.[16,17] All of the solutions presented here were obtained using local time stepping with a Courant number of 0.96. The initial conditions are freestream throughout the domain, and the boundary conditions are applied using an explicit phantom point formulation.[19] Since the problems considered are everywhere supersonic, extrapolated inflow and outflow boundary conditions are used. Zero-pressure-gradient boundary conditions are used for the solid surfaces.

A two-dimensional 5-deg ramp and a two-dimensional 5-deg ramp near a flat plate provide the test cases for comparing the techniques previously discussed. All calculations are made for Mach 2.0 flow. By accurately predicting the flowfield for this test configuration the feasibility of the nonaligned multigrid technique is shown. Solutions generated by the rest of the methods discussed are to show the relative improvement of the nonaligned multigrid and to provide an understanding of how the various components of the nonaligned multigrid work. The algorithms are coded in "C" and the numerical solutions are generated using double precision numbers on a single processor of an Iris 4D/320VGX work station.

Before comparing the efficiency and overall cost of the various methods, two items need to be defined. First, a measure of the amount of work performed on the multiple grid methods needs to be defined to compare with the single grid work. This measure is referred to as a work unit and is actually equivalent to the multigrid work unit[7,8] for a FAS calculation. The number of work units associated with a particular level is the ratio of the number of grid points on the current level to the number of grid points on the largest level (grid with the largest number of points). With this definition the work required for the restriction and prolongation operations for the NAM method and the boundary interpolation for the Schwartz method are neglected. This follows the spirit of the standard multigrid work unit definition and seems reasonable since the cost of interpolation between levels is insignificant compared with the cost of the flow solution procedure. With this definition one work unit is equivalent to one iteration on a single grid with the same number of points as the largest grid in a multiple grid method.

The second item to be defined is a measure of how well the current solution estimate satisfies the numerical scheme. This is accomplished by using the l_2 norm of the residual. In the NAM only the solution on the highest level at any point in the domain is considered to be the correct physical solution; therefore only the residual from the highest level at any point is considered in the residual calculation. For the Schwartz method, on the other hand, all unblanked cells contribute to the residual.

5-deg Ramp Calculations

The methods discussed earlier are first implemented on a 5-deg ramp. First, a single grid solution is generated by applying the FDS to the 40×32 background grid in Fig. 2. Density contours from this first-order solution indicate that the shock is smeared over approximately 20 cells at the back plane.

By adding a 16×16 local grid as indicated in Fig. 2, the accuracy of the solution is dramatically improved. Aligning the local grid lines parallel to the shock wave allows the FDS to produce a much more accurate solution than is possible on the global grid. The density contours from this overset grid solution indicate that the shock is three local grid cells wide at the back plane. This solution is much better than could be obtained on the global grid alone and is virtually the same as the theoretical solution.

The NAM procedure and the Schwartz method both give practically identical results when applied to the system of overset grids depicted in Fig. 2. The Schwartz method is implemented by cutting a hole in the global grid in the vicinity of the shock. For the NAM method, the local grid is designated as level 1 and the global grid as level 2. The best convergence was found to be for a NAM(2-3-3) cycle. This notation simply means that two iterations are made on level 1, followed by three on level 2, and the cycle is completed by three additional iterations on level 1.

Figure 3 shows the residual histories for the global grid only calculation, the Schwartz method, and the NAM calculations. This plot shows that both the Schwartz method calculation (340.4 s) and the NAM calculation (137.6 s) required significantly greater time than the single grid calculation (96.2 s). The NAM calculations require approximately one-third the number of work units required by the Schwartz method and yielded a 59% reduction in CPU time over that required by the Schwartz method.

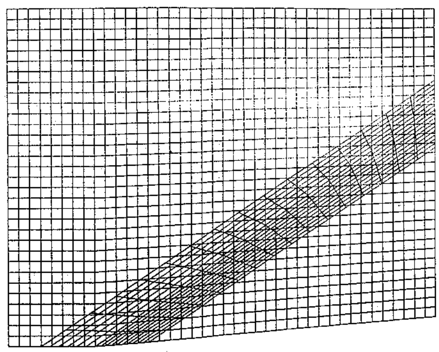

Fig. 2 Overset grids for the 5-deg ramp.

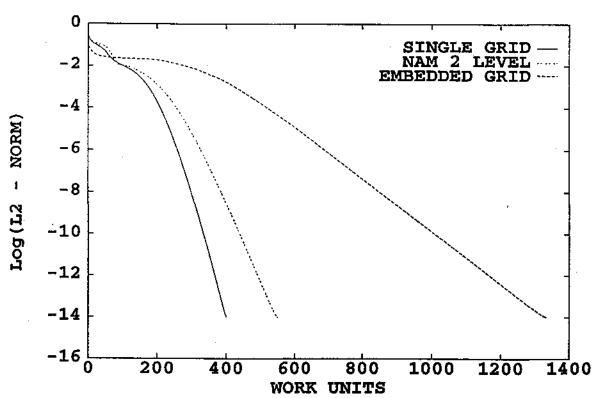

Fig. 3 Residual histories for the 5-deg ramp calculations.

A three-level NAM procedure is defined by making the local grid level 1, the global grid level 2, and coarsening level 2 to make level 3 (a 20 × 16 grid). This system takes 114.3 CPU s to converge to machine zero for a NAM(3-2-2-1-4) cycle. This system requires only 69.7 more work units (5.2 s) than the single grid calculation while producing a far superior solution. This is a 69% reduction in the CPU time required by the Schwartz method.

When the local grid, level 1, is coarsened to make an additional level, only a slight gain in convergence is obtained. This seems to indicate that the most significant gain in convergence will come from simply implementing NAM on the existing overset grid system. Further improvement in convergence is more likely to come from coarsening the global grid rather than the local grid.

5-deg Ramp Near a Flat Plate Calculations

A single grid solution is obtained on the global grid (40 × 32) in Fig. 4a. Density contours from the solution are plotted in Fig. 4b after converging to machine zero. Once again the single grid solution shows the general trend of a highly dissipative first-order method attempting to capture a shock structure.

An overset system of four grids counting the original coarse global grid is used to resolve the shock and reflection. First, a 16 × 16 local grid aligned with the initial shock is inserted (grid 1) as indicated in Fig. 4a. Next grid 2, a 16 × 16 local grid aligned with the reflected shock, is added. Since neither of these could accurately resolve the flow near the shock reflection point, a semicircular grid is added (grid 3). The radial lines in this grid are arranged such that one grid line will coincide with the initial shock and another line with the reflected shock.

Density contours of the solution obtained from both the NAM and the Schwartz method for this system are plotted in Fig. 4c. This new solution is very accurate and compares well with the theoretical solution.

For this case the NAM is applied to the four grid system in Fig. 4a treating the circular grid as level 1, the grid aligned with the initial shock as level 2, the grid aligned with the reflected shock as level 3, and the global grid as level 4. The four level grid scheduling diagram is shown in Fig. 5. The circled numbers indicate the number of iterations performed on each level for a single NAM cycle.

In this case the single (coarse) grid calculation requires only 550 iterations to converge to machine zero using 131.6 CPU s. The residual histories for the single grid, the Schwartz method, and the NAM solution are plotted in Fig. 6. The time required for the Schwartz method is 865.0 CPU s whereas the NAM solution requires 712.5 CPU s, a 17.6% reduction in CPU time.

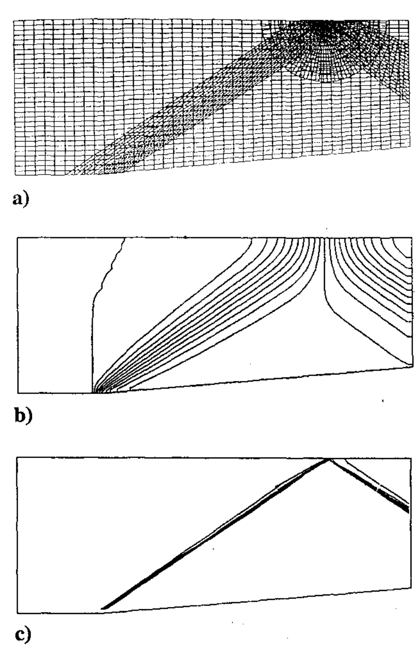

Fig. 4 a) Overset grids for the 5-deg ramp near a flat plate, b) single grid density contours, and c) overset grid density contours.

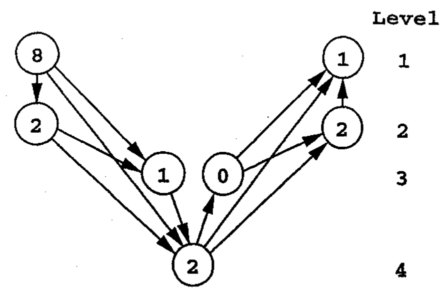

Fig. 5 NAM four level grid schedule.

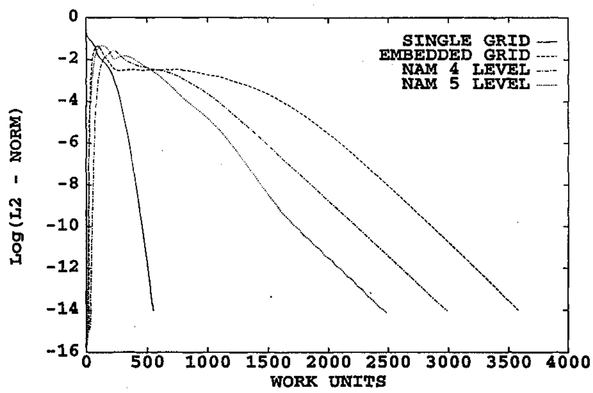

Fig. 6 Residual histories for the 5-deg ramp near a flat plate.

Coarsening the global grid to produce a five level system provides additional savings in the CPU time. A NAM(8-2-2-2-1-1-1-2-1) cycle is used to obtain the solution in only 623.8 CPU s. This is 27.9% faster than the Schwartz method.

Conclusions

The nonaligned multigrid was introduced to provide increased convergence for overset grid systems by treating the individual grids as levels in the multigrid solution procedure. This technique was able to converge 69% faster than the standard boundary point interpolation procedure (Schwartz method) for the 5-deg ramp and 28% faster for a more complex reflected shock case. For these test cases, the largest increase in convergence rate came from applying the nonaligned multigrid method to the communication between overset grids. Further coarsening of the global grid was next in effectiveness, and coarsening of local grids gave little improvement.

References

[1]Dougherty, F. C., "Development of a Chimera Grid Scheme with Applications to Unsteady Problems," Ph.D. Dissertation, Dept. of Aeronautics and Astronautics, Stanford Univ., Palo Alto, CA, June 1985.

[2]Dougherty, F. C., Benek, J. A., and Steger, J. L., "On Applications of Chimera Grid Schemes to Store Separation," NASA-TM-88193, Oct. 1985.

[3]Benek, J. A., Steger, J. L., Dougherty, F. C., and Buning, P. G., "Chimera: A Grid Embedding Technique," Arnold Engineering and Development Center, AEDC-TR-85-64, Arnold AFB, TN, Dec. 1985.

[4]Meakin, R., and Suhs, N., "Unsteady Aerodynamic Simulation of Multiple Bodies in Relative Motion," AIAA Paper 89-1996, June 1989.

[5]Tang, W. P., "Generalized Schwartz Splittings," *SIAM Journal on Scientific and Statistical Computing*, Vol. 13, No. 2, 1992, pp. 573–595.

[6]Johnson, R., "A Multigrid Approach to Embedded-Grid Solvers," Wright Lab., Armament Directorate, WL-TR-92-7039, Eglin AFB, FL, Aug. 1992.

[7]Brandt, A., "Multigrid Techniques: 1984 Guide with Applications to Fluid Dynamics," Computational Fluid Dynamics Lecture Series, von Kármán Inst. for Fluid Dynamics, Rhôde-Saint-Genèse, Belgium, March 1984.

[8]Anderson, W. K., "Implicit Multigrid Algorithms for the Three-Dimensional Flux Split Euler Equations," Ph.D. Dissertation, Dept. of Aerospace Engineering, Mississippi State Univ., Starkville, MS, Aug. 1986.

[9]Hackbusch, W., *Multi-Grid Methods and Applications*, Springer–Verlag, Berlin, 1985.

[10]McCormick, S. F., *Multilevel Adaptive Methods for Partial Differential Equations*, Society for Industrial and Applied Mathematics, Philadelphia, PA, 1989.

[11]Brandt, A., "Multilevel Adaptive Computations in Fluid Dynamics," *AIAA Journal*, Vol. 18, No. 10, 1980, pp. 1165–1172.

[12]Berger, M. J., and Jameson, A., "Automatic Adaptive Grid Refinement for the Euler Equations," *AIAA Journal*, Vol. 23, No. 4, 1985, pp. 561–568.

[13]Baysal, O., Fouladi, K., and Lessard, V. R., "Multigrid and Upwind Viscous Flow Solver on Three-Dimensional Overlapped and Embedded Grids," *AIAA Journal*, Vol. 29, No. 6, 1991, pp. 903–910.

[14]Yadlin, Y., and Caughey, D. A., "Block Multigrid Implicit Solution of the Euler Equations of Compressible Fluid Flow," *AIAA Journal*, Vol. 29, No. 5, 1991, pp. 712–719.

[15]Henshaw, W. D., and Chesshire, G., "Multigrid on Composite Meshes," *SIAM Journal on Scientific and Statistical Computation*, Vol. 8, No. 6, 1987, pp. 914–923.

[16]Roe, P. L., "Approximate Riemann Solvers, Parameter Vectors, and Difference Schemes," *Journal of Computational Physics*, Vol. 43, No. 2, 1981, pp. 357–372.

[17]Whitfield, D. L., Janus, J. M., and Simpson, L. B., "Implicit Finite Volume High Resolution Wave-Split Scheme for Solving the Unsteady Three-Dimensional Euler and Navier–Stokes Equations on Stationary or Dynamic Grids," Mississippi State Univ., Engineering and Industrial Research Station, MSSU-EIRS-ASE-88-2, Starkville, MS, Feb. 1988.

[18]Mavriplis, D. J., "Multigrid Solution of the Two Dimensional Euler Equations on Unstructured Triangular Meshes," *AIAA Journal*, Vol. 26, No. 7, 1988, pp. 824–831.

[19]Whitfield, D. L., and Janus, J. M., "Three-Dimensional Unsteady Euler Equations Solution Using Flux Vector Splitting," AIAA Paper 84-1552, June 1984.